TEST #1

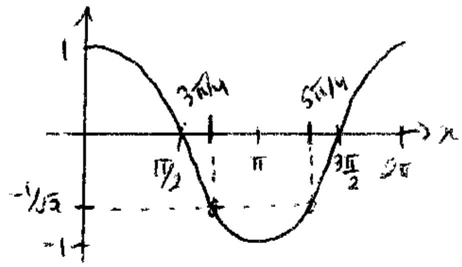
Max = 20

Student Number: $\boxed{\text{Solutions}}$

- Time: 80 min.
- Only basic scientific calculators are permitted: non-programmable, non-graphing, no differentiation or integration capability. Notes or books are not permitted.
- Work all problems in the space provided. Use the backs of the pages for rough work if necessary. Do not use any other paper.
- Write *only* in non-erasable ink (ball-point or pen), not in pencil. Cross out, if necessary, but do not erase or overwrite. Graphs and sketches may be drawn in pencil.
- Problems require complete and clearly presented solutions and carry part marks if there is substantial correct work toward the solution.

A

1. [2 points] Find all values of x in the interval $[0, 2\pi]$ that satisfy $-1 \le \cos x \le \frac{-1}{\sqrt{2}}$.



$\cos x = -1/\sqrt{2}$ if $x = 3\pi/4$ or $5\pi/4$

so $\boxed{3\pi/4 \le x \le 5\pi/4}$

0.5 0.5 0.5 0.5

2. [1 point] What is $\cos(\arcsin x)$?

let $\theta = \arcsin x$

we want $\cos\theta$

but $x = \sin\theta$, so $x^2 + \cos^2\theta = \sin^2\theta + \cos^2\theta = 1$

then $\cos^2\theta = 1 - x^2$

and so $\boxed{\cos\theta = \sqrt{1-x^2}}$ 1

A

3. [2 points] Solve for x: $\ln(\ln(x+4)) = 1$.

$$\ln(\ln(x+4)) = 1$$

so $$e^{\ln(\ln(x+4))} = e^1$$

or $$\ln(x+4) = e$$

then $$e^{\ln(x+4)} = e^e$$

so $$x+4 = e^e$$

$$\therefore \boxed{x = e^e - 4} \approx \boxed{11.1543}$$

← OR →

1

1

4. [3 points] Find a formula for the inverse of $f(x) = e^{e^{x+2}}$.

$$y = e^{e^{x+2}}$$

so $$\ln y = \ln(e^{e^{x+2}}) = e^{x+2}$$

and then $$\ln(\ln y) = \ln(e^{x+2}) = x+2$$

so $$\ln(\ln y) - 2 = x$$

2

switch x and y to get

$$\boxed{y = f^{-1}(x) = \ln(\ln x) - 2}$$ 1

A

5. [4 points] Given the function $f(x) = \cos(2\pi x)$,

(a) find the average rate of change of f on

(i) $[1, 1.1]$

$$\frac{\cos(2\pi(1.1)) - \cos(2\pi)}{1.1 - 1} = \frac{0.8090 - 1}{0.1} = \boxed{-1.9100}$$

1

(ii) $[1, 1.01]$

$$\frac{\cos(2\pi(1.01)) - \cos(2\pi)}{1.01 - 1} = \frac{0.9980 - 1}{0.01} \quad \boxed{-0.2000}$$

1

(iii) $[1, 1.001]$

$$\frac{\cos(2\pi(1.001)) - \cos(2\pi)}{1.001 - 1} = \frac{0.99998 - 1}{0.001} = \boxed{-0.0200}$$

1

(b) estimate the instantaneous rate of change at $x = 1$

$$\boxed{f'(1) = 0}$$ 1

A

6. [4 points] Use the definition of the derivative to find $f'(x)$ if $f(x) = \dfrac{x}{x+2}$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\dfrac{x+h}{x+h+2} - \dfrac{x}{x+2}}{h}$$

$$= \lim_{h \to 0} \frac{1}{h} \left[\frac{x+h}{x+h+2} - \frac{x}{x+2} \right]$$

$$= \lim_{h \to 0} \frac{1}{h} \left[\frac{(x+h)(x+2) - x(x+h+2)}{(x+h+2)(x+2)} \right]$$

$$= \lim_{h \to 0} \frac{1}{h} \left[\frac{\cancel{x^2} + \cancel{xh} + \cancel{2x} + 2h - \cancel{x^2} - \cancel{xh} - \cancel{2x}}{(x+h+2)(x+2)} \right]$$

$$= \lim_{h \to 0} \frac{1}{\cancel{h}} \left[\frac{2\cancel{h}}{(x+h+2)(x+2)} \right]$$

$$= \lim_{h \to 0} \frac{2}{(x+h+2)(x+2)}$$

$$= \boxed{\frac{2}{(x+2)^2}}$$

1

3

A

7. [4 points] Sketch the graph of a function satisfying:

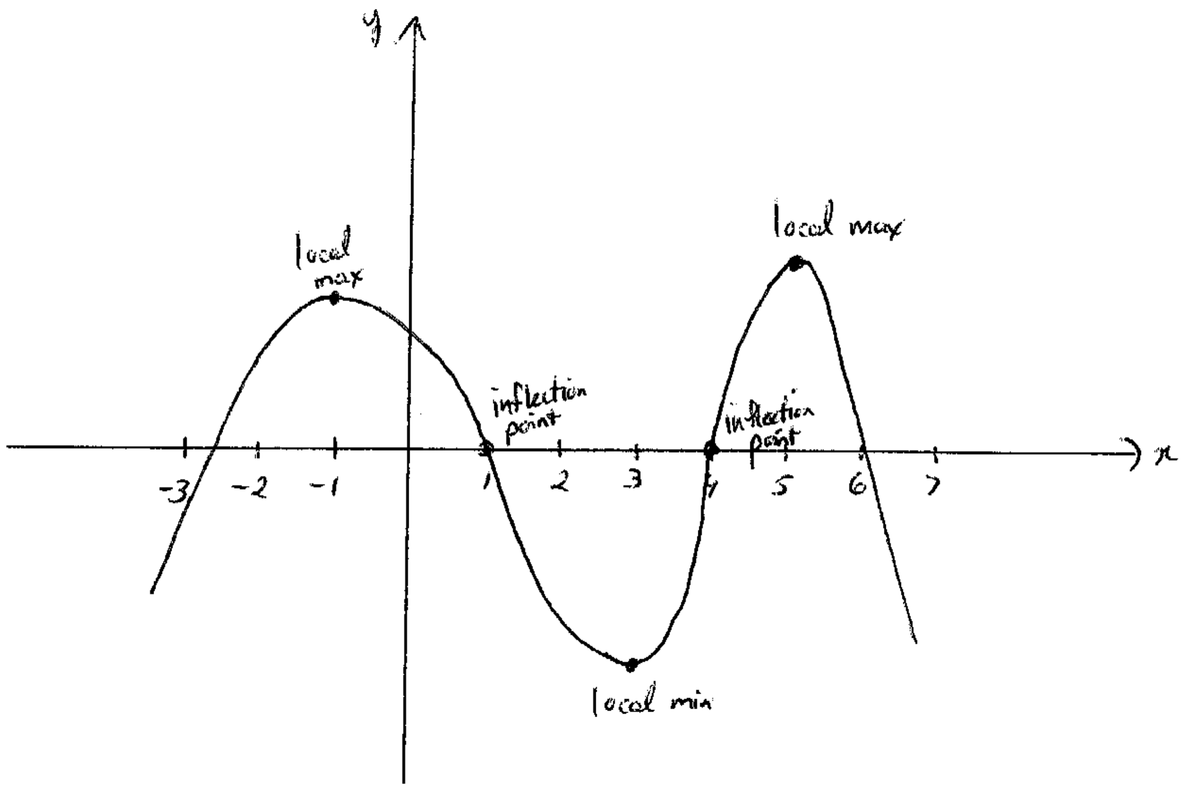
$f'(-1) = f'(3) = f'(5) = 0$

$f'(x) > 0$ if $x < -1$ or $3 < x < 5$

$f'(x) < 0$ if $-1 < x < 3$ or $x > 5$

$f''(x) > 0$ if $1 < x < 4$

$f''(x) < 0$ if $x < 1$ or $x > 4$



(2) for shape

(1) for local extrema in correct places

(1) for inflection pts in correct places

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Student Number: $\boxed{\text{Solution}}$

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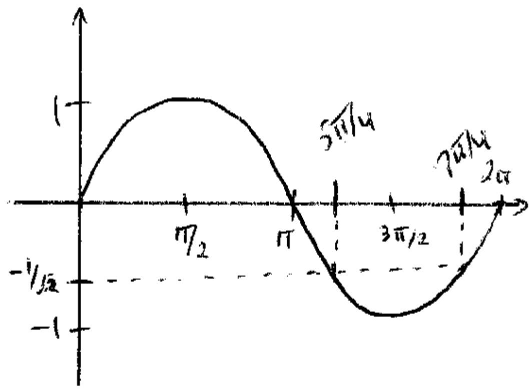
• Problems require complete and clearly presented solutions and carry part marks if there is substantial correct work toward the solution.

(B)

1. [2 points] Find all values of x in the interval $[0, 2\pi]$ that satisfy $-1 \le \sin x \le \frac{-1}{\sqrt{2}}$.



$\sin x = -1/\sqrt{2}$ if $x = 5\pi/4$ or $7\pi/4$

$$\boxed{5\pi/4 \le x \le 7\pi/4}$$

0.5 0.5 0.5 0.5

2. [1 point] What is $\sin(\arccos x)$?

let $\theta = \arccos x$

then we want $\sin \theta$

but $x = \cos\theta$

so $x^2 + \sin^2\theta = \cos^2\theta + \sin^2\theta = 1$

so $\sin^2\theta = 1 - x^2$

$\therefore \boxed{\sin\theta = \sqrt{1-x^2}}$ 1

(B)

3. [2 points] Solve for x: $\ln(\ln(x-3)) = 1$.

$$\ln(\ln(x-3)) = 1$$

so $$e^{\ln(\ln(x-3))} = e^1$$

or $$\ln(x-3) = e$$

then $$e^{\ln(x-3)} = e^e$$

so $$x-3 = e^e$$

ok

$$\therefore \boxed{x = 3+e^e} \simeq \boxed{18.1543}$$

(1) (1)

4. [3 points] Find a formula for the inverse of $f(x) = e^{e^{x-5}}$.

$$y = e^{e^{x-5}}$$

so $$\ln y = \ln(e^{e^{x-5}}) = e^{x-5}$$

then $$\ln(\ln y) = \ln(e^{x-5}) = x-5$$

so $$\ln(\ln y) + 5 = x$$

(2)

switch x and y to get

$$\boxed{y = f^{-1}(x) = \ln(\ln x) + 5}$$ (1)

(B)

5. [4 points] Given the function $f(x) = \sin(2\pi x)$,

(a) find the average rate of change of f on

(i) $[1, 1.1]$

$$\frac{\sin(2\pi(1.1)) - \sin(2\pi)}{1.1 - 1} = \frac{0.58779 - 0}{0.1} = \boxed{5.8779}$$

1

(ii) $[1, 1.01]$

$$\frac{\sin(2\pi(1.01)) - \sin(2\pi)}{1.01 - 1} = \frac{0.06279 - 0}{0.01} = \boxed{6.2790}$$

1

(iii) $[1, 1.001]$

$$\frac{\sin(2\pi(1.001)) - \sin(2\pi)}{1.001 - 1} = \frac{0.006283 - 0}{0.001} = \boxed{6.2830}$$

1

(b) estimate the instantaneous rate of change at $x = 1$

$$\boxed{f'(1) = 6.28}$$ 1

13

6. [4 points] Use the definition of the derivative to find $f'(x)$ if $f(x) = \dfrac{x}{x-1}$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{x+h}{x+h-1} - \frac{x}{x-1}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{(x+h)(x-1) - x(x+h-1)}{(x+h-1)(x-1)}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{\cancel{x^2} + \cancel{xh} - h - \cancel{x} - \cancel{x^2} - \cancel{xh} + \cancel{x}}{(x+h-1)(x-1)}\right]$$

$$= \lim_{h\to 0} \frac{1}{\cancel{h}}\left[\frac{-\cancel{h}}{(x+h-1)(x-1)}\right]$$

$$= \lim_{h\to 0} \frac{-1}{(x+h-1)(x-1)}$$

$$= \boxed{\frac{-1}{(x-1)^2}}$$

3

1

(B)

7. [4 points] Sketch the graph of a function satisfying:

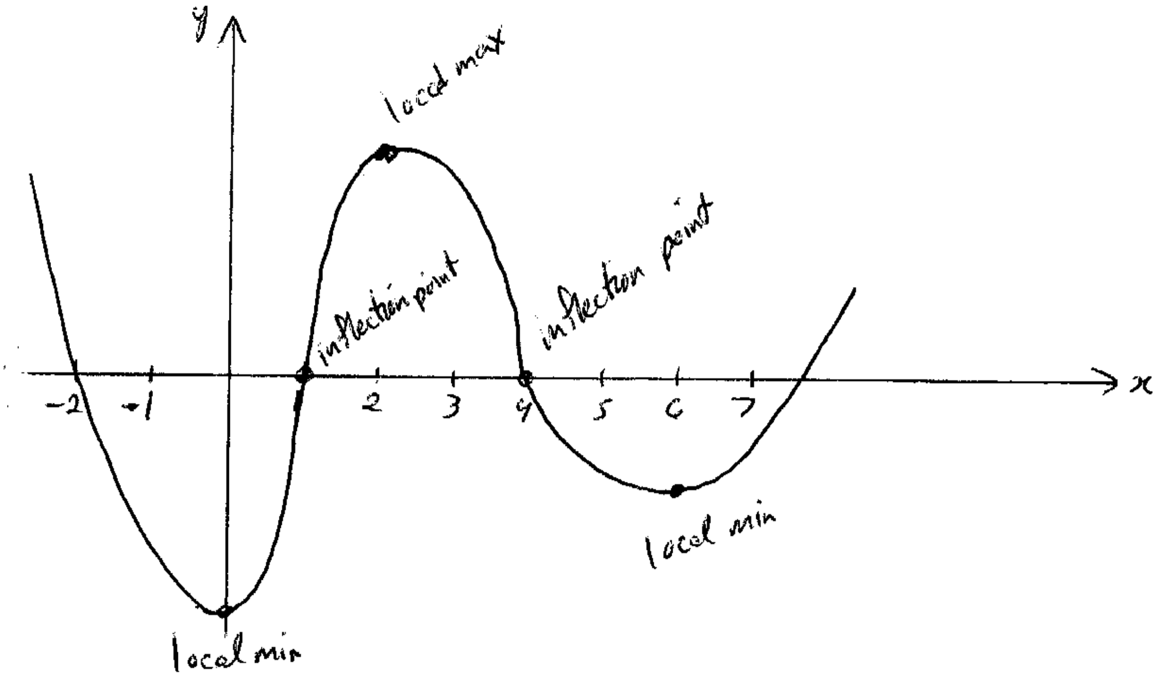
$f'(0) = f'(2) = f'(6) = 0$

$f'(x) > 0$ if $0 < x < 2$ or $x > 6$

$f'(x) < 0$ if $x < 0$ or $2 < x < 6$

$f''(x) > 0$ if $x < 1$ or $x > 4$

$f''(x) < 0$ if $1 < x < 4$



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